Random Processes

Nimit Singhania

October 15, 2022

The focus of this article is Random Processes. Nature is filled with random processes that occur periodically, leading to the world that we admire and relish. From the simplest process of formation of waves in the ocean, to the budding of new leaves on trees, there are random processes involved which are difficult to predict, yet the outcome is concretely observed in the form of waves crashing on the ocean surface, or the tree blooming to be filled with new leaves, bringing a new spirit to the tree. Such processes occur everywhere, whether we notice them or not, and this has been carefully studied by biologists and computer scientists.

The most basic example of a random process is a Markov chain. A Markov chain consists of a number of states in which the system can be. The system proceeds to one state from the other with a certain probability. A state is transient if it is only visited briefly in the life-cycle of the system. It is persistent, if it is visited periodically or the chain concludes by the system resting in this state. The system of weather in a place could be described as a Markov chain, where the seasons represent states, and the transition from one to the other is representated by probability of the weather transitioning from one state to the other, or remaining in the current state. If a season arrives early, then the weather transitions sooner into the season than later. Many such examples can be given to illustrate Markov chains, and are often used to describe natural and biological phenomenan.

Many interesting properties have been studied about Markov chains, for example, the time it takes to go from one state to the other, or the time it takes to cover all states in the system. Such properties can be easily computed in expectation, and provide useful information about the system. Further, based on the composition of the system, we can further determine the eventual states in which the system spends most time, and their eventual probabilities. This further provides the composition of the system once it reaches its final (non-transient) states. A state is considered absorbing if the system remains in this state and does not transition to other states. If such a state is present within the system and there is a small probability of transitioning to this state from all other states, then the system will converge to the absorbing state. On the other hand, if there is a set of periodic states, that the system can repeatedly visit, none of the states need to be absorbing.

An extention of Markov chains is that of Markov decision processes, where random processes are mixed with decision making and the next state in the system is determined both by the decision at the current state and the random process that is undertaken from the decision. Such processes are prevelant in controlled processes, like chemical reactions, human-controlled biological synthesis etc. The rate of reaction in a nuclear reactor is determined by the exposure of carbon rods which is controlled by the humans monitoring the reaction. A Markov decision process is often associated with an optimization goal, i.e. to identify the best decision in each state of the system that leads to optimal result. If a unique decision is available at each state, then there is a stationary strategy for the system and the process itself becomes a Markov chain. A stationary strategy assigns a unique decision to each state and use the same action when the state is observed. For example, when rate of nuclear reaction is higher than desired, the exposure of carbon rods is increased, while when it is lower, the exposure is decreased. This is a staionary strategy and can be performed automatically.

With booming data-driven systems, we are often surrounded by random processes. These random processes can be difficult to predict, and therefore, need significant analysis and consideration. The eventual result of such processes needs to be studied and learnt from examples – this can be non-trivial task for humans who are part of such processes. The hope is that these random processes will converge to stationary strategies and convergent states, which make them easier to assess and work with in the real-world.